Zick Method Adaptation for Horizontal Cylindrical Vessels (H.C.V.) Asymmetrically Supported on Three Saddle Supports with Support Settlements

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This paper presents a way of using the Zick method for the design of the horizontal cylindrical vessel supported in three points. The effect of the settlements has been considered taking into account the PD 5500-2009 prescriptions. The hypothesis used and the results of the calculations reached the conclusion that the bounded of a cylindrical vessel is not recommended to be made in three points.

Keywords: horizontal cylindrical vessel supported on three saddle supports, settlements, Zick method

In a previous paper [1], there was presented the way Zick method had been extended to the h.c.v. asymmetrically supported on three saddle supports without support settlements, as described under the British Standards P.D. 5500 - 2003 [2].

Hereunder, we intend to consider also the effect of support settlements on the status of stresses in the recipient shell.

Sectional stresses

The h.c.v. supported on three saddle supports, as treated in the Beam Theory, are once statically undetermined. To calculate the bending moment on the intermediary support, the "Equation of the Three Moments" for continuous beams shall be applied [3, 4].

In case of support settlement, either due to the variable settlement of the foundation land, or due to faulty execution of works, it is necessary to consider these settlements in the equation of the three moments [4].

Equation of the Three Moments



If v_{12} , v_{2} and v_{3} are the settlements in the three supports (fig. 1), the equation of the three moments has to be completed with the following member:

$$6 EI\left(\frac{v_1 - v_2}{L_{12}} + \frac{v_3 - v_2}{L_{23}}\right)$$
(1)

The modelling of h.c.v. supported on three saddle supports, compliant with the schematization proposed by L. P. Zick for h.c.v. supported on two saddle supports, is presented in figure 2 (equivalent of the scheme in figure G.3.3-2 in paper [2]). As far as possible, the notations were kept similar to those in PD 5500 - 2009 [2].

Considering the support settlements, the equation of the three moments (as per assumption of constant thickness of the cylindrical shell) becomes as follows:

$$M_{4}^{'}L_{12} + 2M_{4}^{''}(L_{12} + L_{23}) + M_{4}^{''}L_{23} +$$

+ $m_{21}L_{12} + m_{23}L_{23} + 6 \operatorname{EI}\left(\frac{v_{1} - v_{2}}{L_{12}} + \frac{v_{3} - v_{2}}{L_{23}}\right)$ (2)

where:

$$M_4' = \frac{W}{4} \left(r^2 - b^2 \right) - \frac{WA_1}{2}$$
(3)

$$M_4''' = \frac{w}{4} (r^2 - b^2) - \frac{w A_2^2}{2}$$
(4)

$$m_{21} = \frac{WL_{12}^2}{4}$$
(5)

$$m_{23} = \frac{wL_{23}}{4}$$
(6)

where:

 m_{21} and m_{23} – are the uniformly distributed load w for the segment between supports 1-2 and 2-3; v_1, v_2, v_3 – settlements in the supports 1, 2, 3;

E -longitudinal elasticity module of the recipient shell; $I = \pi r^2 t$ – moment of inertia of the shell.

The meaning of the other units is deducted from figure 2.

The bending moment M_{A} on the support 2 can be calculated by using equation (2).

Using this value and making elementary calculations, it is possible to determine reactions W_1 , W_2 and W_3 by making use of the following equations (fig. 2b):

$$W_{1} = w \left(\frac{2}{3}b + A_{1} + \frac{L_{12}}{2}\right) - \frac{M_{4} - M_{4}}{L_{12}}$$
(7)

$$W_{2} = w \frac{L_{12} + L_{23}}{2} + \frac{M_{4} - M_{4}}{L_{12}} + \frac{M_{4} - M_{4}}{L_{23}}$$
(8)

$$W_{3} = w \left(\frac{2}{3}b + A_{2} + \frac{L_{23}}{2}\right) - \frac{M_{4} - M_{4}}{L_{23}}$$
(9)

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Fig. 2.

The shear forces T_1^{RT} , T_2^{LT} , T_2^{RT} and T_3^{LT} shall be calculated by using the equations below:

$$T_1^{RT} = w_1 - w \left(\frac{2}{3}b + A_1\right) = w \frac{L_{12}}{2} - \frac{M_4^{-1} - M_4}{L_{12}}$$
 (10)

$$T_2^{LT} = T_1^{RT} - wL_{12} = -w \frac{L_{12}}{2} - \frac{M_4 - M_4}{L_{12}}$$
(11)

$$T_2^{RT} = T_2^{LT} + w_2 = w \frac{L_{23}}{2} - \frac{M_4^{\prime\prime} - M_4^{\prime\prime\prime}}{L_{23}}$$
 (12)

$$T_3^{LT} = T_2^{RT} - wL_{23} = -w \frac{L_{23}}{2} - \frac{M_4 - M_4}{L_{23}} = -W_3 + w \left(\frac{2}{3}b + A_2\right)$$
(13)

The extreme bending moments M_3 and M_3 , corresponding to the intervals between supports 1-2 and 2-3 respectively, are calculated according to the following equations:

$$M_{3} = M_{4} + \frac{(T_{1}^{RT})^{2}}{2w} = M_{4}^{"} + \frac{(T_{2}^{LT})^{2}}{2w}$$
(14)

$$M_{3}^{"} = M_{4}^{"} + \frac{(T_{2}^{RT})^{2}}{2w} = M_{4}^{"} + \frac{(T_{3}^{LT})^{2}}{2w}$$
(15)

Stresses

The adaptation of Zick method, as per the design formulae provided in the British Standards P.D. 5500 – 2009

[2], to the analysed case requires the highlighting of the changes in the equivalent equations for two supports. We shall observe the order provisioned in the Standards.

Longitudinal stress against the maximum moment between supports

There shall be selected the maximum value of the bending moment M_3 and M_3 . Stresses f_1 and f_2 shall be determined by replacing this value of the bending moment M_3 in the equations (G.3.3-5) and (G.3.3-6) from P.D. 5500 – 2009 [2].

Longitudinal stress against the saddle supports

There will be selected the highest value of the bending moments M_4 , M_4 , and M_4 . If this value corresponds to the bending moments on

If this value corresponds to the bending moments on end supports (M_4 ', M_4 '''), then the possibility of shell reinforcement by heads (ends) may be considered (if $A_1/A_2 \le r/2$).

² Stresses f_3 and f_4 shall be calculated by replacing the highest value chosen for moment M_4 from the equations (G.3.3-7) and (G.3.3-8) from P.D. 5500 – 2009 [2].

For coefficients K_1 and K_2 [4], the following equations shall be used:

$$K_{1,2} \le \frac{r}{2}$$
 $K_1 = K_2 = 1$ (16)

 $A_{1,2} > r/2$ or for the intermediate support

A

$$K_{1} = \frac{\Delta + \sin \Delta \cdot \cos \Delta - 2 \frac{\sin^{2} \Delta}{\Delta}}{\pi \left(\frac{\sin \Delta}{\Delta} - \cos \Delta\right)}$$
(17)

$$K_{2} = \frac{\Delta + \sin\Delta \cdot \cos\Delta - 2\frac{\sin^{2}\Delta}{\Delta}}{\pi \left(1 - \frac{\sin\Delta}{\Delta}\right)}$$
(18)

where [4]:

$$\Delta = \frac{\pi}{180} \left(\frac{5\theta}{12} + 30 \right) \text{ [rad]} \tag{19}$$

Angle θ is in sexagesimale degree.

Tangential shear stress

There shall be selected the highest absolute value of the shear forces T_1^{RT} , T_2^{LT} , T_2^{RT} and T_3^{LT} , determined using the equations (10) ... (13) and it shall be noted as T_{max} .

Considering the way of setting up the equation of the tangential shear stress under paper [5], equation (G.3.3-9) from [2] becomes:

$$q = -\frac{K_3 \cdot T_{max}}{r \cdot t}$$
(20)

For the other cases mentioned under P.D. 5500 – 2009 [2], the same equations shall be used, taking into account that the highest value W_1, W_2 or W_3 , determined according to equations (7) \dots (9), shall be used for W₁.

Circumferential stress

There shall be selected the highest value of reactions W_1, W_2 or W_3 , which shall be used instead of the reaction occurring in the equations corresponding to circumferential stress in Chapter G.3.3.2.6 in P.D. 5500 -2009 [2].

Considering the specifications expressed by Zick in the 1971 edition of the paper [5] (page 964), the following equation will be used for L:

 $L = L_{12} + L_{23}$.

Consequently, the stresses $f_6 \dots f_8$ shall be calculated by the same equations from the normative PD 5500-2009 [2].

Verification of effective stresses as against to the allowable values shall be done in the same manner as in the normative.

Calculus example

The goal of the example is to analyse the effect of the settlements in the bending moments from the supports of a horizontal cylindrical vessel bounded on three supports (the values $M_4^{'}$, $M_4^{''}$ and $M_4^{'''}$ from the figure 2). The calculus scheme of the vessel is presented in the

figure 3.

If the weight of the recipient and of the hydrostatic test fluid are considered, an uniform pressure w = 123.58 N/ mm is obtained.

The numerical calculations have been performed both with classical method of strength of materials and with finite elements method (FEM).

In order to analyse the effect of the settlements for the middle support a vertical displacement ($\Delta_{2}=5$ mm) has been considered.

The ANSYS program has been used and the cylindrical vessel has been meshed in 32 PIPE finite elements.

The following cases have been analysed:

- cylindrical vessel supported in 3 points without any settlements;

-cylindrical vessel supported in 3 points with some settlements of ± 5 and ± 10 mm for the middle supports;

-cylindrical vessel supported in 3 points with some settlements of ± 5 and ± 10 mm for one of the end supports; -cylindrical vessel supported only in 2 points (without

the middle support);

-cylindrical vessel supported in 2 points (without an end support)

After analysing the above mentioned cases the following results have been obtained:

- the bending moments from the end supported points (because of the equal length of the cantilevers) have the values: $M_4 = M_4$ " = - 1.704 x 10⁸ N. mm; - the values of the bending moments from the middle

supported point, when a vertical settlement (0,-5,+5,-10, +10)mm for the middle support has been considered, are presented in table 1. The values of the bending



(21)

 $\Delta_1 = \Delta_3 = 0$

No.	Δ_3	M4''	$ M_{4}'' _{i}$
i=	[mm]	[N·mm]	$\overline{ M_4'' _1}$
1	0	-1.928·10 ⁸	1
2	-5	-1.103·10 ¹⁰	57.210
3	+5	1.065.1010	55.239
4	-10	-2.187·10 ¹⁰	113.434
5	+10	$2.148 \cdot 10^{10}$	111.411

Table 2

moments from the middle supported point for the same values of the settlements have been considered (for an end support) and are presented in the table 1;

- if the middle support is eliminated the bending moment from the middle point has the value $M_3 = 1.081 \times 10^9$ N.mm, the vertical displacement from this point being only 0.3 mm;

- the fraction between the bending moments from middle point in the case that the tank is supported only in two points(at the ends) and the bending moment from the same middle point in the case that the tank is supported in three points (without any settlements) has the value: 5.607;

- in the hypothesis that an end support is eliminated the bending moment in the middle supported point has the value: M_a = -2.629 x 10⁹N.mm

Such à situation is not recommended to be used in design techniques.

Conclusions

The situation of bounded the horizontal cylindrical vessel in three supported points is not recommended because of possible settlements that have the disadvantage of conducing at the middle support to high values of the bending moments (tables 1 and 2), The analysed

The analysed cylindrical vessel had a high enough bending rigidity in order to be supported only in two extreme points.

References

 $\Delta_1 = \Delta_2 = 0$

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